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Letter

On the Nonequivalency of "Right" and "Left" in the Electrodynamics of Moving Bodies

Answer to M. Podlaha's Objections [*International Journal of Theoretical Physics*, **11**, 69 (1974)] against our Theory Presented in "The Axiomatic Foundations of the Theory of Special Relativity" [*International Journal of Theoretical Physics*, **5**, 403 (1972)]

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1. In 1974 M. Podlaha criticized our theory presented in the paper "The Axiomatic Foundations of the Theory of Special Relativity" (Stiegler, 1972). The present paper gives detailed arguments against Podlaha's objections and shows (a) that no other function than $\lambda(v)$ can be introduced if we start from the system of axioms A_1–A_4 given in our previous paper; (b) that Podlaha's main objections have their origin in the misunderstanding of the function $\lambda(v)$, and in particular that the introduction of two functions $\lambda(v)$ and $\mu(v)$ with $\lambda(v) \neq \mu(v)$, as is done by Podlaha, contradicts the principle of causality; (c) that from the uniqueness of $\lambda(v)$ following from the axioms A_1–A_4 it results that changing the direction of the relative velocity $v \to -v$—i.e., changing the "right" and the "left," the intensity of the vector of the electric as well as the vector of the magnetic field in S' will be changed—that is, in the electrodynamics of moving bodies based on axioms A_1–A_4 the "right" and

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the "left" are physically not equivalent, contrary to the assertion of Podlaha. This theoretical result can be decided only by experiments. If "right" and "left" in the electrodynamics of moving bodies are physically not equivalent, then by measurements performed in the reference system S' one should observe a difference of the energy density of the electromagnetic field in accordance with relation

$$\Delta u_{s'} = u_{s'}(-v) - u_{s'}(v) = \frac{(v/c)}{\pi[1 - (v^2/c^2)]}(E_y H_z - E_z H_y) \tag{1'}$$

which follows from the expression for energy density of the electromagnetic field in S', the relations (6) and (7) of the present paper and the Lorentz transformation, i.e. the relation (3.3.2) of our paper (Stiegler 1972), in the case $\lambda = 1$ corresponding to the case of acceptance of Einstein's "postulate of symmetry" A_5. In the case that experiments would agree with (1′) we would have *creatio ex nihilo*, since the energy density of the electromagnetic field at the change $v \to -v$ would increase, in contradiction to the principle of energy. In the second case, where experiments performed in S' would agree with $\Delta u_{s'} = 0$, the "right" and "left" would be physically equivalent in the electrodynamics of moving bodies, but this experiment would contradict the Lorentz transformation, which conduces to the relation (1′). The question of the possible violation of the principle of indiscernibility of "right" and "left" in the electrodynamics of moving bodies can be decided only by experiments. Such an experiment would represent an *experimentum crucis* not only for electrodynamics of moving bodies, but also for the theory of special relativity.

2. From our system of axioms A_1–A_4 and the assumption concerning the origins of Galileian systems of reference given on p. 410 of our paper it follows that

$$x_i' = f_i(v, x_0, x_1, x_2, x_3) = [\lambda(v)]^{1/2}(e_k a_{ik} x_k) \tag{3.1.35}$$

and in the special case, as given on p. 412, the relation (3.3.2) and not the

$$x' = \frac{1}{[\lambda(v)]^{1/2}} k(x + vt)$$

$$y' = \frac{1}{[\lambda(v)]^{1/2}} y$$

$$z' = \frac{1}{[\lambda(v)]^{1/2}} z$$

$$t' = \frac{1}{[\lambda(v)]^{1/2}} k\left(t + \frac{vx}{c^2}\right)$$

relation as Podlaha asserts. Thus the objection (I) of Podlaha is false. We shall now prove that the objections (II) and (III) of Podlaha contradict the principle of causality. According to our theory (Stiegler, 1972) the transformation of coordinates and times connecting the Galileian systems of reference S and S', respectively, is given by

$$\begin{aligned} x' &= [\lambda(v)]^{1/2}k(x - vt) \\ y' &= [\lambda(v)]^{1/2}y \\ z' &= [\lambda(v)]^{1/2}z \\ t' &= [\lambda(v)]^{1/2}k(t - vx/c^2) \end{aligned} \qquad k = \frac{1}{(1 - v^2/c^2)^{1/2}}$$

Let S'' be a third Galileian system of reference such that X'' coincides with the X and X' axes. Then in an analogous way we have

$$\begin{aligned} x'' &= [\mu(v)]^{1/2}k(x' - vt') \\ y'' &= [\mu(v)]^{1/2}y' \\ z'' &= [\mu(v)]^{1/2}z' \\ t'' &= [\mu(v)]^{1/2}k(t' - vx'/c^2) \end{aligned} \qquad k = \frac{1}{(1 - v^2/c^2)^{1/2}}$$

as transformation connecting the coordinates and times of the systems S' and S''. We assert that the relation

$$\lambda(v) \neq \mu(v)$$

which, according to Podlaha, must exist, is in contradiction with the principle of causality. This will be obvious from the following consideration. Let an electromagnetic wave in S having the frequency $\nu = \nu_0$ be propagated in direction $\alpha = \alpha_0$, $\beta = \beta_0$, $\gamma = \gamma_0$. Then in accordance with Section 3.5, p. 414 of our paper (Stiegler, 1972), the observer in the "moving" system S' will observe

$$\begin{aligned} \nu' &= \frac{1}{[\lambda(v)]^{1/2}}\nu_0 \frac{1 - (v/c)\cos\alpha_0}{(1 - v^2/c^2)^{1/2}}, \qquad \cos\alpha' = \frac{\cos\alpha_0 - v/c}{1 - (v/c)\cos\alpha_0} \\ \cos\beta' &= \frac{(\cos\beta_0)(1 - v^2/c^2)^{1/2}}{1 - (v/c)\cos\alpha_0}, \qquad \cos\gamma' = \frac{(\cos\gamma_0)(1 - v^2/c^2)^{1/2}}{1 - (v/c)\cos\alpha_0} \end{aligned} \tag{1}$$

We shall now repeat the experiment under identical physical conditions: Let an electromagnetic wave in S' having the frequency $\nu' = \nu_0$ be propagated in S' in the direction $\alpha' = \alpha_0$, $\beta' = \beta_0$, $\gamma' = \gamma_0$. Then in accordance with (1) and Section 3.5 of our paper the observer in the moving system S'' must find

$$\begin{aligned} \nu'' &= \frac{1}{[\mu(v)]^{1/2}}\nu_0 \frac{1 - (v/c)\cos\alpha_0}{(1 - v^2/c^2)^{1/2}}, \qquad \cos\alpha'' = \frac{\cos\alpha_0 - v/c}{1 - (v/c)\cos\alpha_0} \\ \cos\beta'' &= \frac{(\cos\beta_0)(1 - v^2/c^2)^{1/2}}{1 - (v/c)\cos\alpha_0}, \qquad \cos\gamma'' = \frac{(\cos\gamma_0)(1 - v^2/c^2)^{1/2}}{1 - (v/c)\cos\alpha_0} \end{aligned} \tag{2}$$

If the two functions $\lambda(v)$ and $\mu(v)$ were different, i.e.,

$$\lambda(v) \neq \mu(v) \tag{3}$$

then, the observer of the moving reference system in the first experiment would observe another frequency than the observer in the second experiment: $\nu' \neq \nu''$, although both experiments are supposed to be performed under physically identical conditions! This would contradict to the principle of causality valid at the macrocosmical level, in accordance with which from the same "cause" under the same physical conditions there must follow the same "effect." Thus the assumption of Podlaha—i.e., relation (3), is not possible. And objections (II) and (III) of Podlaha are obviously false.

3. For the next considerations the following explications will be useful. Let the origin O' (of S') move relative to the origin O (of S) along the X axis in the sense of increasing (decreasing) x values. Then we shall say that O' moves on the "right" ("left") relative to O. In the same way if the origin O moves relative to O' along the X' axis in the sense of increasing (decreasing) x' values, then we shall say that O moves on the "right" ("left") relative to O'. To the motion on the "right" ("left") there corresponds the velocity $+v$ $(-v)$. Denote by $v_{ss'}$ the velocity of O' as measured from O and by $v_{s's}$ the velocity of the origin O as measured from O'. Then, in accordance to the axiom A_4 of our mentioned paper we have

$$v_{ss'} = -v_{s's} \tag{4}$$

Denoting further "increasing" ("decreasing") by the symbols $\uparrow$ ($\downarrow$), then the following scheme is valid:

$v_{ss'}$	$v_{s's}$
$\uparrow$, ("right"), $+v$	$\downarrow$, ("left"), $-v$
$\downarrow$, ("left"), $-v$	$\uparrow$, ("right"), $+v$

From the above scheme there results the following: If the velocity of O' relative to O is changed in accordance with $v \rightarrow -v$, i.e., if for an observer in O the "right" and the "left" are changed, then, using (4), the velocity of O relative to O' will be changed in accordance with $v \rightarrow -v$, i.e., for the observer in O' the "left" will be changed into the "right."

4. Concerning the objection (IV) of Podlaha the following must be said. According to Podlaha the Thornedyke experiment leads to the conclusion that the function $[\lambda(v)]^{1/2}$ has in the case of the transition $S \rightarrow S'$ the form

$$[\lambda(v)]^{1/2} = \frac{1}{(1 - v^2/c^2)^{1/2+\gamma}} \tag{5}$$

Podlaha's assertion that $\lambda(v) = \lambda(-v)$ is a pure mathematical conclusion, which follows from the relation (5) arbitrarily proposed by Podlaha, since the Thornedyke experiment does not give any information about the nature of the function $\lambda(v)$ at the change

$$v \rightarrow -v$$

If Podlaha were consistent, then the relation between $\lambda(v)$ and $\lambda(-v)$ as well as between $\gamma(v)$ and $\gamma(-v)$ should be decided only by the Thornedyke experiment in the case of the change $v \rightarrow -v$. Thus the objection (IV) of Podlaha is not acceptable. Finally from the uniqueness of $\lambda(v)$, the relations (3.6.8) and (3.6.15) and the fundamental relation $\lambda(v)\cdot\lambda(-v) = 1$ [equation (3.6.12)] of our paper it results that changing the direction of the relative velocity $v \rightarrow -v$, i.e., changing the "left" and the "right," the transformation law of any component of the vector of the electric and magnetic field will be changed:

$$\begin{aligned} X'(v) &= [\lambda(v)]^{1/2}X & L'(v) &= [\lambda(v)]^{1/2}L \\ Y'(v) &= [\lambda(v)]^{1/2}k[Y - (v/c)N] & M'(v) &= [\lambda(v)]^{1/2}k[M + (v/c)Z] \\ Z'(v) &= [\lambda(v)]^{1/2}k[Z + (v/c)M] & N'(v) &= [\lambda(v)]^{1/2}k[N - (v/c)Y] \end{aligned}$$

and

$$\begin{aligned} X'(-v) &= \frac{1}{[\lambda(v)]^{1/2}}X & L'(-v) &= \frac{1}{[\lambda(v)]^{1/2}}L \\ Y'(-v) &= \frac{1}{[\lambda(v)]^{1/2}}k[Y + (v/c)N] & M'(-v) &= \frac{1}{[\lambda(v)]^{1/2}}k[M - (v/c)Z] \\ Z'(-v) &= \frac{1}{[\lambda(v)]^{1/2}}k[Z - (v/c)M] & N'(-v) &= \frac{1}{[\lambda(v)]^{1/2}}k[N + (v/c)Y] \end{aligned}$$

$$k = \frac{1}{[1 - v^2/c^2]^{1/2}}$$

from which for the quadrate of the vector of the electric field as well for the vector of the magnetic field in S' we get

$$\begin{aligned} \mathfrak{E}'^2(v) &= X'^2(v) + Y'^2(v) + Z'^2(v) \\ &= \lambda(v)\left[X^2 + k^2\left(Y - \frac{v}{c}N\right)^2 + k^2\left(Z + \frac{v}{c}M\right)^2\right] \\ \mathfrak{E}'^2(-v) &= X'^2(-v) + Y'^2(-v) + Z'^2(-v) \\ &= \frac{1}{[\lambda(v)]^{1/2}}\left[X^2 + k^2\left(Y + \frac{v}{c}N\right)^2 + k^2\left(Z - \frac{v}{c}M\right)^2\right] \end{aligned}$$

and

$$\mathfrak{E}'^2(v) \neq \mathfrak{E}'^2(-v) \tag{6}$$

as well

$$\mathfrak{H}'^2(v) = L'^2(v) + M'^2(v) + N'^2(v)$$
$$= \lambda(v)\left[L^2 + k^2\left(M + \frac{v}{c}Z\right)^2 + k^2\left(N - \frac{v}{c}Y\right)^2\right]$$
$$\mathfrak{H}'^2(-v) = L'^2(-v) + M'^2(-v) + N'^2(-v)$$
$$= \frac{1}{[\lambda(v)]^{1/2}}\left[L^2 + k^2\left(M - \frac{v}{c}Z\right)^2 + k^2\left(N + \frac{v}{c}Y\right)^2\right]$$

and

$$\mathfrak{H}'^2(v) \neq \mathfrak{H}'^2(-v) \tag{7}$$

respectively.

It is very important to point out that the relations (6) and (7) hold in the case of acceptance of the validity of Einstein's "axiom of symmetry" A_5, where $\lambda(v) = 1$ (see pp. 413 and 418 of our above-cited paper) as well as in the case of nonacceptance of A_5, where $\lambda(v)$ could perhaps be $\neq 1$.

Thus from the uniqueness of $\lambda(v)$ following from the axioms A_1–A_5 it results that changing the direction of the relative velocity

$$v \rightarrow -v$$

i.e., changing the "left" and the "right," the intensity of the vector of the electric as well as the magnetic field will be changed. This result does not depend on the acceptance or nonacceptance of Einstein's "postulate of symmetry" A_5 where $\lambda(v)$ is equal to 1 or is generally different from 1, respectively. Our considerations lead to the fundamental result: In the electrodynamics of moving bodies based on the system of axioms A_1–A_4 or A_1–A_5 [which corresponds to the Einstein case as presented in Einstein (1905)] the "right" and the "left" are physically not equivalent, or what is the same, in the electrodynamics of moving bodies one must distinguish between "right" and "left," contrary to the assertion of Podlaha.

5. The question whether the intensity of the vector of the electric and magnetic field in the "moving" system of reference S' will be really changed in accordance with (6) and (7), respectively, where $\lambda(v)$ could perhaps be $\neq 1$ (in the case of the validity of the system of axioms A_1–A_4) or $=1$ (the latter will be surely satisfied in the case of the validity of Einstein's "axiom of symmetry" A_5) if the direction of the relative velocity

$$v \rightarrow -v$$

or, what is the same, "right" and "left" will be changed, can be decided only by experiments. The realization of such experiments would be most desirable and of the greatest importance, since they would represent a new "experimentum crucis" not only for the special theory of relativity but at the same time for the electrodynamics of moving bodies. The energy density of the electromagnetic field in the reference system S' is given by (Ivanenko-Sokolov, 1953)

$$u_{s'}(v) = \frac{1}{8\pi}[\mathfrak{E}'^2(v) + \mathfrak{H}'^2(v)] \tag{8}$$

Changing "right" and "left" in accordance with $v \to -v$ and taking into account the expressions for $\mathfrak{E}'^2(v)$ in (6) and $\mathfrak{H}'^2(v)$ in (7) we get in the case $\lambda = 1$ (Lorentz transformation) for the difference of the energy density of the electromagnetic field in the reference system S' the expression

$$\Delta u_{s'} = u_{s'}(-v) - u_{s'}(v) = \frac{1}{\pi}\frac{v/c}{(1-(v^2/c^2))}(E_y H_z - E_z H_y) \tag{9}$$

If "right" and "left" in the electrodynamics of moving bodies are physically not equivalent, then by a measurement of the energy density of the electromagnetic field in S' a difference in S' should be observed in accordance with (9), which is a consequence of Lorentz transformation, i.e. of the relation (3.3.2) of our paper (Stiegler, 1972), in the case $\lambda = 1$ corresponding to the acceptance of Einstein's "postulate of symmetry" A_5. In the case that experiments would agree with (9)—or equivalently with Lorentz transformation—we would have *creatio ex nihilo,* since the energy density would increase, in contradiction with the principle of energy. In the second case, where experiments performed in S' would agree with $\Delta u_{s'} = 0$, the "right" and "left" in the electrodynamics of moving bodies would be physically equivalent. But this experimental result would contradict the Lorentz transformation which conduces to the relation (9).

6. Our theoretical result has nothing to do with the violation of the principle of parity in weak interactions proved by C. N. Yang and T. D. Lee (1956), since in the electrodynamics of moving bodies, valid at macrocosmical level the violation of the principle of indiscernibility of "right" and "left" does relate exclusively to the change of the intensity of the vector of the electric and of the magnetic field connected with the change of direction of relative velocity of Galileian systems of reference S and S'.

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